RIEMANN SURFACES AND THE THETA FUNCTION

BY

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Introduction

The purpose of this paper is to present a clear exposition of certain theorems of Riemann, notably Theorem 8 below, which he obtained from his study of the θ function as a means of solving the Jacobi inversion problem. A perusal of Riemann's collected works, [4], shows that this was a topic of great interest to him. For this paper, one may consult [4], pp. 133–142, 212–224, 487–504, and the Supplement, pp. 1–59. Many mathematicians, in the half century after Riemann, tried to elucidate and justify his results. In this connection we may mention Christoffel, Noether, Weber, Rost, and Poincaré. Citations of the older literature may be found in the books, [2], [3], and [6].

Despite all these efforts, it is difficult for me to say whether or not complete proofs have been given to everything that has been claimed. In this paper, we hope to give correct proofs of some of these interesting results, along with some new theorems. Our method is essentially that of Riemann and his followers, although the language may be slightly more modern. The key to our method is consideration of the role of the base point, i.e., lower limit of the integrals of first kind, and its influence on the vector K of Riemann constants. The roles of the base point and K seem to have been overlooked by all, probably because of the statement of Riemann, [4], p. 133 and p. 213, that, under a suitable normalization, the vector K vanishes. Finally, having available the concept of an abstract Riemann surface gives one a distinct advantage over being tied down to a particular branched covering of the sphere.

In the first section, we prove the basic theorem concerning the zeros of certain "multiplicative functions". On the whole, in this section, we try to conform with the

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