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Wiener regularity for large solutions of nonlinear equations

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1. Introduction

This paper concerns *large solutions* to nonlinear elliptic equations in arbitrary bounded domains $\Omega\subset\mathbf{R}^n$, $n\geq 3$. These are solutions $u\in C^2_{\mathrm{loc}}(\Omega)$ to the nonlinear problem

$$\begin{cases} \Delta u - |u|^{q-1}u = 0 & \text{in } \Omega, \\ u(x)\to+\infty, & \text{when } x\to\partial\Omega. \end{cases} \tag{1.1}$$

For the parameter q we always assume in this paper that

$$q>1. \tag{1.2}$$

Equation (1.1) is the model equation for a broad class of semilinear elliptic problems admitting comparison principle. Apart from the importance for partial differential equations, interest in large solutions in general domains comes from two different sources: the theory of spatial branching processes and conformal differential geometry. Of the two basic questions concerning problem (1.1) in arbitrary domains Ω—namely, existence and uniqueness—our main result completely resolves the first. Theorem 1.1 states that the solubility of (1.1) is equivalent to a Wiener-type test with respect to a certain capacity. As to the second question, it is well known that uniqueness for (1.1) fails in general domains [39], [14], [29]. Note that the strong maximum principle for elliptic equations implies that u from (1.1) satisfies

$$u>0, \quad \Delta u - u^q = 0 \text{ in } \Omega. \tag{1.3}$$

Hence without loss of generality we need to consider only positive solutions of (1.1).

After the ground-breaking papers by Perkins [67], Dynkin [19], and Le Gall [45], solutions of (1.1) and (1.3) attracted a lot of attention from probabilists. Currently