On polyharmonic continuation by reflection formulas

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1. Introduction and summary*

Let Ω be an open connected set in $\mathbf{R}^n$, which is contained in the half space $\mathbf{R}^n_+=\{x\colon x_1\geqq 0\}$, and let an open connected subset ω of the boundary of Ω be situated in the hyperplane $x_1=0$. Then $\Omega\cup\omega$ is open in $\mathbf{R}^n_+$. A p-harmonic function in Ω is a $2p$ times differentiable solution of the equation

$$\Delta^p u = 0, \quad u\in C^{2p}(\Omega), \tag{1.1}$$

where $\Delta=\sum_{i=1}^n \frac{\partial^2}{\partial x_i^2}$ is the Laplace operator. We denote the set of all such functions by $H^p(\Omega)$. It will be seen that if $u\in H^p(\Omega)$, u is in fact analytic in Ω. We shall consider functions $u\in H^p(\Omega)$ which also satisfy a set of p boundary conditions

$$\lim_{x_1\to +0} q_i(D_1)u(x_1, x') = 0,\ (0, x')\in\omega, \quad i = 1, \ldots, p, \tag{1.2}$$

where $q_i(D_1)$ are linearly independent polynomials in $D_1=\frac{\partial}{\partial x_1}$, with constant coefficients and x' denotes $(x_2, \ldots, x_n)$. In (1.2) we do not suppose x' to be fixed as $x_1\to +0$. We shall also use the notation $q_i(D_1)u(x)=o(1)$ as $x_1\to +0$. It will be shown that these functions can be continued as polyharmonic functions across ω into the half space $\mathbf{R}^n_-=\{x\colon x_1<0\}$. Very general theorems of this type have been given by Hörmander in [7], where he considers solutions of general elliptic and hypoelliptic differential equations with constant coefficients.

* The main part of this work constituted a PH.D. thesis accepted at Stockholm University 1973.