On the adjoint of an elliptic linear differential operator and its potential theory

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1. Introduction

In her thesis [4], R.-M. Hervé develops Brelot's axiomatic potential theory. Within this theory she constructs an adjoint potential theory satisfying the same axioms. She applies this to the potential theory associated with an elliptic linear second-order differential operator L. When the adjoint operator L^* exists in the classical sense and has Hölder-continuous coefficients, the adjoint potential theory coincides with that of L^*. In Section 3 of this paper we generalize this fact to the case when the coefficients of L are assumed to be locally α-Hölder continuous and L^* is defined in the sense of distributions. This result easily implies some properties of supersolutions of the equation $L^*u = 0$ proved by Littman [5]. He shows that they satisfy a minimum principle and have some approximation properties.

Under the same assumptions, we prove in Section 4 that the distribution solutions of $L^*u = 0$ are locally α-Hölder continuous. In Section 5 we obtain a formula for Hervé's L^*-harmonic measure of a domain ω. This measure is shown to have an area density given simply by a conormal derivative of the Green's function of L in ω. Finally, we prove a Fredholm type theorem for the Dirichlet problems for L and L^* in a given domain.

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2. Preliminaries

Suppose we are given a domain $\Omega_0 \subset \mathbf{R}^n$, $n \geq 2$, and a differential operator

$$Lu = a^{ij}u_{ij} + b^i u_i + cu,$$