Spectral analysis in weighted L^1 spaces on $\mathbf{R}$

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Introduction

Weighted L^1 algebras on $\mathbf{R}$ were introduced by Beurling in [1]. A Beurling algebra L_p^1 is defined as the convolution algebra of (equivalence classes of) functions f, Lebesgue measurable on $\mathbf{R}$ and satisfying

$$\|f\|_p = \int_{\mathbf{R}} |f(x)|p(x)dx < \infty$$

where p is the weight-function associated with the algebra in question. In order that L_p^1 be an algebra, a condition of the type

$$p(x+y) \leq p(x)p(y)$$

has to be fulfilled by p. According to the size of p, Beurling talks of different cases. If, for simplicity, we assume p to be even, we consider the limit

$$\alpha = \lim_{|x|\to\infty} \frac{\log p(x)}{|x|}$$

which can be shown to exist. If $\alpha > 0$, we have the *analytic* case. When $\alpha = 0$, the *quasi-analytic* and *non-quasianalytic* cases are distinguished according as

$$\int_{\mathbf{R}} \frac{\log p(x)}{1+x^2} dx$$

diverges or converges, respectively.

A central problem in the study of any Banach algebra is that of its ideal structure, in particular the problem of *spectral analysis*. We say that spectral analysis holds in an algebra B if every closed (proper) ideal in B is contained in a regular maximal ideal of B. The General Tauberian Theorem of Wiener [15] tells us that spectral analysis does hold in ordinary $L^1(\mathbf{R})$, and this result has been extended to Beurling's