On an extremal configuration for capacity

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1. Main theorem

It is well known that the capacity of a closed set E on the (unit) circle is decreased by circular symmetrization [1, pp. 31—36]. Thus, if the length mE of E is L, we have the estimate $\operatorname{cap} E \geqq \sin(L/4)$ [1, p. 35]. How large can $\operatorname{cap} E$ be, if $mE=L$ and E consists of a given number of arcs, n arcs? The maximal configuration is given by a set E^* of n arcs of equal length, L/n, "regularly" or "symmetrically" distributed around the circle.

Theorem 1. *Let* $E^*=\bigcup_{k=0}^{n-1}\{\exp(i\vartheta)\colon\ -L/2n \leqq \vartheta-2\pi k/n \leqq L/2n\}$ *and let E be a union of n arcs on the unit circle of total length L. Then*

$$\operatorname{cap} E \leqq \operatorname{cap} E^* = (\sin(L/4))^{1/n}, \tag{1}$$

with equality for $E=E^*$.

A proof of this theorem follows from work of Dubinin's [2]. He proved a conjecture by Gončar for harmonic measure by introducing a process called desymmetrization, which can also be used for transforming E^* to E and for comparing the capacities of these sets.

In terms of equivalent characteristics of E and E^*, the inequality in Theorem 1 can be stated for *Robin constants,* (see [1, p. 30]), γ of E and γ^* of E^*, as

$$\gamma^* \leqq \gamma, \tag{1'}$$

and for *reduced extremal distances,* (see [1, pp. 78—80]), as

$$\delta(0, E^*) \leqq \delta(0, E). \tag{1''}$$

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