Space analogues of some theorems for subharmonic and meromorphic functions

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1. Introduction

Denote points in n dimensional Euclidean space $\mathbf{R}^n$, $n \geqq 3$, by $x=(x_1, x_2, \ldots, x_n)$. Let $r=|x|$ and $x_1=r\cos\theta$, $0\leqq\theta\leqq\pi$. For $r>0$ let $B(r)=\{x:|x|<r\}$, $S(r)=\{x:|x|=r\}$, and $S=S(1)$. For $0\leqq\alpha\leqq\pi$, let $C(\alpha)=S\cap\{x:\theta<\alpha\}$. If E is a set contained in $S(r)$, let ∂E denote the boundary of E relative to $S(r)$. Let H^m denote m dimensional Hausdorff measure on $\mathbf{R}^n$.

If f is defined on a set $E\subset\mathbf{R}^n$, let $\theta(r)$ for $0<r<\infty$ be defined by

$$H^{n-1}(C(\theta(r))) = H^{n-1}(p(S(r)\cap E))$$

where p denotes the radial projection of $\mathbf{R}^n-\{0\}$ onto S. For $0\leqq\theta\leqq\theta(r)$, let

$$\hat{f}(r,\theta) = \sup \int_F f(ry)\,dH^{n-1}y,$$

where the supremum is taken over all measurable sets $F\subset p(S(r)\cap E)$ with

$$H^{n-1}(F) = H^{n-1}(C(\theta)).$$

Given a set $E\subset[0,\infty)$, let

$$\overline{\log\operatorname{dens}}\, E = \limsup_{r\to\infty}\left(\int_{E\cap(1,r)}\frac{dt}{t}\Big/\log r\right)$$

$$\underline{\log\operatorname{dens}}\, E = \liminf_{r\to\infty}\left(\int_{E\cap(1,r)}\frac{dt}{t}\Big/\log r\right).$$

Let u be equal H^n almost every where on $\mathbf{R}^n$ to the difference of two subharmonic functions. By the Riesz representation theorem there is associated with this difference a unique signed Borel measure ν whose total variation on compact sets is finite. Let $\nu=\nu^+-\nu^-$ denote the Jordan decomposition of ν. To simplify matters, we will assume that $\nu^+(B(1))=0$ or equivalently that u is equal H^n almost everywhere in $B(1)$ to a subharmonic function.