Removable singularities of CR-functions

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0. Introduction, statement and discussion of the results. There are well-known theorems concerning removable singularities of analytic or harmonic functions from various classes in planar domains, see for example [11]. There are also generalizations to higher-dimensional domains and general elliptic differential operators instead of the $\bar{\partial}$ or Laplace operator [4]. The description of removable singularities depends on the class of functions and is usually given in terms of capacity or Hausdorff measure. For operators appearing in the theory of several complex variables such as the Cauchy—Riemann system or the $\bar{\partial}_b$ operator (the boundary Cauchy—Riemann operator for smooth domains in $\mathbf{C}^n$) we have to expect new phenomena so that the complete description cannot be given in the terms mentioned. This is suggested, for example, by the well-known Hartogs theorem ([5], Theorem 2.3.2, [8], 16.3.6): every function f analytic in the connected set $\Omega\setminus E$, Ω being a domain in $\mathbf{C}^n$ $(n>1)$ and E a compact subset of Ω, is the restriction to $\Omega\setminus E$ of a function analytic in the whole of Ω. So for the Cauchy—Riemann system a "removable set of singularities" E is not necessarily small in measure or capacity, it can even have a nonempty interior.

Suppose now that a closed set E is situated in $\operatorname{Clos}\Omega$ (not necessarily in Ω!). What the Hartogs theorem suggests in this case is that the removability of E (with respect to the class of all functions analytic in $\Omega\setminus E$) depends only on the behaviour of E near the boundary $\operatorname{Fr}\Omega$ or maybe depends only on $E\cap\operatorname{Fr}\Omega$ ($\operatorname{Clos} A$ means the closure of the set A, $\operatorname{Fr} A$ its boundary). This fact must imply Hartogs type theorems for the boundary Cauchy—Riemann operator. That means that we have to expect the existence of sets $A\subset\operatorname{Fr}\Omega$ which are removable singularities for the boundary Cauchy—Riemann operator and are "large" in some sense. So they are not necessarily removable for arbitrary differential operators of first order.

Now we shall give precise statements of the mentioned results. In the statement and proof of the results we restrict ourselves to the case of the unit ball $B(=B^n)$ in $\mathbf{C}^n$ although it is not hard to see that the main results are true for strictly pseudo-