Some good unirational families of space curves

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The problem of classifying space curves, which goes back to Halphen and Noether, may be roughly divided into 3 parts:

(i) list the maximal families (i.e. Hilbert scheme components) of smooth curves in $\mathbf{P}^3$;
(ii) describe the properties of a general member Y of a given maximal family (e.g. number of moduli, vanishing of $H^1(N_Y)$, maximal rank etc.);
(iii) describe Y "explicitly" (e.g. by equations).

To date, little progress has been realized on part (i), while part (ii) has been answered in many cases. Part (iii), on the other hand has been answered, apart from curves which are (or almost are) complete intersections, only in a handful of cases. This is in part explained by the results of Harris and Mumford [6] showing that for g large, M_g, the moduli space of curves of genus g, is not unirational; hence a curve of genus g with general moduli cannot be described by equations depending on free parameters.

The purpose of this paper is to add some further cases to the list of those for which part (iii) above has a positive answer. Namely we prove the following result.

Theorem. *For* $d \leqq 15$, $5d-55 \leqq g \leqq 2d-9$, *and* $(d, g) \neq (13, 11)$, *the Hilbert scheme* $H_{d,g}$ *has a unirational component H such that the curve Y corresponding to a general point in H is linearly normal, of maximal rank, has* μ_0 *of maximal rank and* $H^1(N_Y)=0$.

The case $(d, g)=(13, 11)$ is still open.

The cases $(d, g)=(12, 10)$, $(11, 9)$ of the Theorem are proved using the method of monads, as in [3], [4]. The idea is to construct a curve Y with good properties,

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