A shortcut to weighted representation formulas for holomorphic functions

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0. Introduction

In [4] a method was given for generating weighted solution kernels to the $\bar\partial$-equation, i.e. kernels K such that

$$\bar\partial \int K \wedge w = w, \quad \text{if} \quad \bar\partial w = 0.$$

As a by-product a variety of projection kernels (P such that $f = \int Pf$, for f holomorphic) were obtained. These kernels give representation formulas for holomorphic functions which in general consist of an integral over the whole domain and a boundary integral. The projection part and the corresponding representation formulas have proved to be quite fruitful. They have been used by several authors (see e.g. [2], [3], [5] and [10]) to obtain explicit solutions to division and interpolation problems.

The purpose of this paper is to give a short proof of a generalization of the representation formulas in [3] and [4] without making the détour to the $\bar\partial$-problem and the kernels K.

We derive in § 1 a quite general formula (Theorem 1) which is then turned into a more tangible one for bounded domains (Theorem 2). Using logarithmic residues we also obtain weighted versions of certain formulas in [13] and [15]. In § 2 we give a few examples and comments.

To motivate what follows, let us take a brief look at the case $n=1$. Let f be holomorphic in a domain $\Omega \subset \mathbf{C}$ and suppose that $Q \in C^1(\bar\Omega \times \bar\Omega)$. We then have

$$\int_\Omega f \frac{\partial Q}{\partial \bar\zeta} \, d\bar\zeta \wedge d\zeta = \int_{\partial\Omega} fQ \, d\zeta$$

and by the Cauchy formula it follows that

$$f(z) = \frac{1}{2\pi i} \int_\Omega f(\zeta) \frac{\partial Q}{\partial \bar\zeta}(\zeta, z) \, d\bar\zeta \wedge d\zeta + \frac{1}{2\pi i} \int_{\partial\Omega} f(\zeta) \frac{1+(z-\zeta)Q(\zeta, z)}{\zeta - z} \, d\zeta.$$