On the equivalence between locally polar and globally polar sets for plurisubharmonic functions on $\mathbb{C}^n$

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We shall prove that a locally polar set in $\mathbb{C}^n$ is globally polar which generalizes a well-known result from potential theory for subharmonic functions and answers a question posed by Lelong [2]. Our method differs from the ones frequently used in potential theory, since it seems that there is a lack in the representation of plurisubharmonic functions by kernels, and the main step in our proof is to find, to every given function which is analytic in a ball, polynomials which are sufficiently small on the set where the given function is small (Proposition). From this the theorem will follow (Lemma 3) because locally a plurisubharmonic function is a Hartogs function. A consequence of the theorem is that an analytic set is globally polar and the theorem also has applications in the theory for capacities and extremal functions in $\mathbb{C}^n$. See for example Siciak [3].

**Definition.** A set $D \subset \mathbb{C}^n$ is called *locally polar* if there exist, to every $z \in D$, an open set $V_z \subset \mathbb{C}^n$ and $u_z \in PSH(V_z)$, where $PSH(V_z)$ denotes the set of all plurisubharmonic functions in $V_z$, so that $z \in V_z$ and such that $u_z|_{V_z \cap D}$, the restriction of $u_z$ to $V_z \cap D$, is equal to $-\infty$. $D$ is *globally polar* if we can take $V_z = \mathbb{C}^n$. For details see [2].

We shall give $\mathbb{C}^n$ the sup-norm and we shall let $\mathcal{A}(V)$, where $V \subset \mathbb{C}^n$ is open, denote the set of all analytic functions on $V$. We note that $f$ has a Taylor series expansion $f(z) = \sum a_r z^r$ if $f \in \mathcal{A}(B(0, S))$, where $B(0, S)$ is the open ball in $\mathbb{C}^n$ with centre 0 and radius $S$, $a_r \in \mathbb{C}$, $r = (r_1, \ldots, r_n)$ is a multi-index and $z^r = z_1^{r_1} \cdots z_n^{r_n}$ where $z = (z_1, \ldots, z_n) \in \mathbb{C}^n$.

**Theorem.** A set $D \subset \mathbb{C}^n$ is globally polar if and only if $D$ is locally polar.

From the theorem we obviously have the following,

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