Ideal theory on non-orientable Klein surfaces

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§ 0. Introduction

That non-empty, connected, non-orientable compact surfaces Y, with non-empty boundary ∂Y, can bear a structure, a dianalytic structure, which allows one to define the notion of analytic "functions" on them has been known essentially since Klein's 1882 monograph [6]. However, only recently have the standard algebra $A(Y)$ of all continuous "functions" on Y that are analytic on $Y \setminus \partial Y \equiv Y^\circ$, and the algebra $H^\infty(Y^\circ)$ of all bounded analytic "functions" on Y°, been studied by Alling, Campbell, and Greenleaf [2, 3, 4, 5]. In these papers it has been shown that $A(Y)$ is a real Banach algebra which does not admit a complex scalar multiplication, whose maximal ideal space is Y, and whose Silov boundary is ∂Y. Further it has been shown that $A(Y)$ is a hypo-Dirichlet algebra whose deficiency is $c - 1$, c being the first Betti number of Y, and that $A^{-1}(Y)/\exp A(Y)$, the factor group of units modulo exponentials, is isomorphic to $\mathbf{Z}_2 \oplus \mathbf{Z}^{c-1}$.

The purpose of this paper is the study of the closed ideals of $A(Y)$. Y admits an unramified double covering morphism p of a compact bordered Riemann surface X such that $p^{-1}(\partial Y) = \partial X$, and X admits an antianalytic involution τ that commutes with p. This *orienting double* (X, p, τ) of Y is unique up to an analytic isomorphism. Let $A(X)$ be the standard algebra on X and for f in $A(X)$, let $\sigma(f) \equiv \varkappa \circ f \circ \tau$, $\varkappa$ being complex conjugation. Then σ is an $\mathbf{R}$-automorphism of $A(X)$ of period 2 that is an isometry. $A(Y)$ is naturally $\mathbf{R}$-isomorphic and

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