Interpolation between weighted L^p-spaces

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1. Introduction

One of the main problems in any application of interpolation space theory is the identification of the interpolation space $(\mathcal{X}_0, \mathcal{X}_1)_\Phi$ generated from a pair of Banach spaces $\mathcal{X}_0, \mathcal{X}_1$ by a given interpolation method Φ. The importance of interpolation space theory stems in part from the frequency with which $(\mathcal{X}_0, \mathcal{X}_1)_\Phi$ is identified with classical Banach spaces and the deeper understanding the theory then brings to these spaces. Applications to Lorentz L^{pq}-spaces and to Lipschitz spaces are good examples (cf. [4], for example).

In this paper the interpolation spaces $(L^p_{\omega_0}, L^p_{\omega_1})_{\theta,q}$ generated between weighted L^p-spaces by the (real) J-, K-methods of Peetre will be characterized. Without real loss of generality we shall assume $\omega_0 \equiv 1$. The »diagonal» spaces $(L^p_{\omega_0}, L^p_{\omega_1})_{\theta,p}$, it is known already, coincide with another weighted L^p-space:

$$(L^p_{\omega_0}, L^p_{\omega_1})_{\theta,p} = L^p_\nu, \quad \nu = \omega_0^{1-\theta}\omega_1^\theta, \quad 0 < \theta < 1$$

([7], [15]) and the associated interpolation theorem reduces to the Stein-Weiss extension of the classical M. Riesz theorem to spaces with changes of measure. Peetre began the characterization of the »off-diagonal» cases by identifying $(L^p_{\omega_0}, L^p_{\omega_1})_{\theta,1}$, $1 < p < \infty$, with one of a family of spaces introduced by Beurling ([3]) in connection with problems of spectral synthesis. Herz later generalized Beurling's definition and considerably clarified Beurling's paper though without systematic recourse to interpolation space theory ([11]). In sections 4 and 5 we complete the characterizations of $(L^p_{\omega_0}, L^p_{\omega_1})_{\theta,q}$, $1 \leq q \leq \infty$, and identifications with the Beurling spaces. With these characterizations as well as the theory of (homogeneous) Besov spaces the various results of Herz can be obtained very easily using simple interpolation space techniques (see [6]).

The Beurling spaces have been considered in many contexts other than spectral synthesis (cf. [9], [10], [12], [13], [18]) with various characterizations being obtained