On thc Valiron deficiencies of integral functions of infinite order

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1. Introduction

Let $f(z)$ be meromorphic in the plane. We define in the normal way the order ϱ and the characteristic $T(r, f)$ of $f(z)$ and also the quantities $m(r, a)$ and $N(r, a)$ for any a in the closed plane.[1])

The Valiron deficiency is defined to be

$$\Delta(a) = \overline{\lim_{r\to\infty}} \frac{m(r, a)}{T(r, f)} = 1 - \underline{\lim_{r\to\infty}} \frac{N(r, a)}{T(r, f)} .$$

We are concerned in this paper with the question of how large the set of a can be for which $\Delta(a, f) > 0$ [3, problem 1.2]. For functions of finite order this problem has recently been completely solved by Hyllengren. He proved [4, Theorem 1] the following

Theorem A. *Let E be any plane point set. Then the following two conditions are equivalent*

a) *There exists a positive number k and an infinite sequence $a_1, a_2, \ldots$ of complex numbers, so that each $a \in E$ satisfies the inequality*

$$|a - a_n| < \exp\{- \exp(nk)\}$$

for infinitely many n.

b) *There exists a real number x, $0 < x < 1$ and a meromorphic function $f(z)$ of finite order in $|z| < \infty$, so that*

$$\Delta(a, f) > x$$

for every a in E.

In fact $f(z)$ can be chosen to be an integral function.

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[1]) for the notation see e.g. [5, p. 158].