# Representations of tensor algebras as quotients of group algebras


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## § 0. Introduction

1. Tensor algebras, or to be precise, projective tensor products of $C(K)$-spaces have important relations both with Hilbert space and with $l^1$. The relation with Hilbert space was discovered by Grothendieck, and was called by him »the fundamental theorem on the metric theory of tensor products». It certainly is the deepest result in this metric theory [see e.g. 6]. The relations with $l^1$ were discovered by Varopoulos, and their importance lies in the fact that they relate the algebra structure of tensor algebras to the algebra structure of group algebras [8]. These relations are two-fold, in the first place, a group algebra can in a canonical way be embedded as a closed subalgebra of a tensor algebra. Through this embedding, information on tensor algebras can be obtained from information on group algebras. In the second place a tensor algebra can be represented as a quotient of a group algebra, so that information on tensor algebras can be transferred to group algebras. The main result in the second connection is the following; if $\{K_i\}_{i=1}^n$ are disjoint compact subsets of a compact abelian group, and if $\bigcup K_i$ is a Kronecker set (or a $K_p$-set), then $A(\sum K_i)$ is a tensor algebra. In this paper we shall consider to what extent the Kronecker condition in the theorem of Varopoulos can be replaced by Helson conditions on the sets. Our main results are the following.

THEOREM A. *To every natural number $n \geq 2$, there corresponds a real number $\alpha_n$, such that if $\{K_i\}_{i=1}^n$ are disjoint compact subsets of a compact abelian group, and if $\bigcup K_i$ is a Helson-$(1-\alpha)$ set, $\alpha < \alpha_n$, then $A(\sum K_i)$ is a tensor algebra.*

THEOREM B. *To every natural number $q \geq 2$, and every natural number $n \geq 2$, there corresponds a real number $\alpha_{q,n}$ such that if $\{K_i\}_{i=1}^n$ are disjoint compact subsets*