Distributions with bounded potentials and absolutely convergent Fourier series

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1. Introduction

In the first part of this paper we show that the capacity of a compact subset of the n-dimensional Euclidean space can be characterized by means of distributions (in the sense of L. Schwartz) which are carried by the set and which have bounded potentials.

In the second part we consider compact subsets of the real line with the property that no non-trivial function can locally be in the class of Fourier transforms of L^1-functions and yet be constant on the intervals of the complement of the set. Using the result from the first part we shall show that a sufficient condition for a set to have this property is that it has logarithmic capacity zero. This improves a result by Kahane and Katznelson [4, p. 21] concerning Cantor sets.

It will also be shown that a necessary condition is that the set has capacity zero with respect to all kernels $(\log^+ 1/|x|)^{2+\delta}$, $\delta > 0$.

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2. Notations and definitions

We denote the n-dimensional Euclidean space by $\mathbf{R}^n$, its points by $x = (x_1, \ldots, x_n)$ and we write $|x| = (x_1^2 + \ldots + x_n^2)^{1/2}$.

By A^{loc} we mean the space of all functions f on $\mathbf{R}^n$ with the property that for each $x \in \mathbf{R}^n$ there exists a neighbourhood V of x and a function g, whose Fourier transform is an integrable function, so that $f = g$ in V.

The Fourier transform of a function, measure or tempered distribution S is denoted by $\hat{S}$.