On strong Ditkin sets

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Let G be a locally compact abelian group with character group Γ, and let $M(G)$ be the convolution algebra consisting of all bounded regular measures on G. The Fourier transform of a measure μ in $M(G)$ is defined by

$$\hat{\mu}(\gamma) = \int_G (x, -\gamma) d\mu(x) \quad (\gamma \in \Gamma) .$$

We shall regard the group algebra $L^1(G)$ as a closed ideal in $M(G)$ (see [9, p. 16]). For a given closed subset E of Γ, let us denote by:

$I(E) = \{f \in L^1(G) : \hat{f} = 0 \text{ on } E\}$;

$I_0(E) = \{f \in L^1(G) : \hat{f} = 0 \text{ on some neighborhood of } E\}$;

$J(E) =$ the closure of $I_0(E)$,

and, for any measure μ in $M(G)$, define

$$\|\mu\|_E = \sup \{\|f * \mu\| : f \in I_0(E),\ \|f\| \leq 1\} .$$

In other words, $\|\mu\|_E$ is the operator norm of the mapping: $f \to f * \mu$ (from $I_0(E)$ into $L^1(G)$).

Definition 1. (cf. [10] and [8]). We say that a closed subset E of Γ is a *Wik set* if there exists a family $\{\mu_\alpha \in M(G)\}_{\alpha \in A}$ of measures which is directed, in the sense that the index set A is a directed set, such that:

(a) $\sup \{\|\mu_\alpha\|_E : \alpha \in A\} < \infty$;

(b) $\hat{\mu}_\alpha(\gamma) \xrightarrow[\alpha \in A]{} 0$ if $\gamma \in E$, and $\hat{\mu}_\alpha(\gamma) \xrightarrow[\alpha \in A]{} 1$ if $\gamma \in E^c$.

Definition 2. (cf. [10]). E is called a *strong Ditkin set* if there exists a directed family $\{\mu_\alpha \in M(G)\}_{\alpha \in A}$ of measures such that: