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Some results on embedding Stein spaces with interpolation

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1. Introduction

In this article we give some remarks on embeddings of Stein manifolds and more generally Stein spaces into $\mathbf{C}^n$ with interpolation. This means that given a Stein manifold X together with a countable discrete subset $A=\{a_1,a_2,a_3,...\}$ in X and a countable discrete subset $E=\{e_1,e_2,e_3,...\}$ in $\mathbf{C}^n$ we try to construct a proper holomorphic embedding

$$\varphi\colon X \hookrightarrow \mathbf{C}^N \quad \text{with} \quad \varphi(a_i)=e_i,\ i\in\mathbf{N}. \tag{1.1}$$

According to results of Eliashberg, Gromov [3] and Schürmann [10] every Stein manifold of dimension $n>1$ can be properly holomorphically embedded into $\mathbf{C}^N$ for all $N\geq n+\left[\frac{1}{2}n\right]+1$. Examples of Forster show that this bound is sharp in general [6]. The question of embedding with interpolation was earlier studied by Prezelj in [8]. Her main result is that for a Stein manifold of dimension $n>1$ the embedding with interpolation is always possible into affine space of dimension greater than or equal to $n+\left[\frac{1}{2}(n+1)\right]+1$, thus needing one extra dimension for odd n compared to the general embedding dimension. The proof of Prezelj follows the method of Eliashberg and Gromov, taking care of the interpolation condition at any step. This method of "desingularization" has the disadvantage that even though one has a Stein manifold which can be embedded into affine space of lower dimension, the construction yields an embedding only into dimension $n+\left[\frac{1}{2}(n+1)\right]+1$. For embeddings of the unit disc into $\mathbf{C}^2$ the problem of interpolation was solved by Globevnik in [7], and can be solved for embeddings of $\mathbf{C}$ into $\mathbf{C}^2$ by the methods of that paper and [2]. The question of interpolation on more general subvarieties is considered by Acquistapace, Broglia and Tognoli in [1].

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