Every sequence converging to O weakly in L_2 contains an unconditional convergence sequence[1]

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The aim of this paper is to prove the above statement, which is clearly equivalent to the following:

THEOREM. *For every sequence of measurable functions* f_n *with*

$$\int f_n^2 \leqq K \quad (n = 1, 2 \ldots)$$

there is a subsequence g_n *and a square integrable function* g *such that the sequence* $h_n = g_n - g$ *is an unconditional convergence sequence.*

Recall that a sequence h_n is called a convergence sequence, if the series $\Sigma c_n h_n$ is convergent almost everywhere, whenever the sequence c_n of real numbers satisfies $\Sigma c_n^2 < \infty$. The sequence h_n is called an unconditional convergence sequence, if every rearrangement of h_n is a convergence sequence. (E.g. the sequence r_n (on $[0, 1]$) of Rademacher functions is known to be an unconditional convergence sequence; while the sequence $\sqrt{2/\pi} \cdot \cos(nx)$ (on $[0, \pi]$) is a convergence sequence (Carleson), but — being a complete orthonormal sequence — it is not an unconditional covergence sequence.)

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[1] Throughout the paper all functions are measurable functions on some measure space $\{X, \mathcal{S}, \mu\}$. It is clear that it is sufficient to prove our Theorem in case of finite measure, thus we can take $\mu(X) = 1$.

As a rule, we do not indicate
the arguments of functions: writing φ, f etc. instead of $\varphi(x), f(x)$ etc., and $\mu(f > \lambda)$ instead of $\mu(\{x; f(x) > \lambda\})$,
and the measure: writing $\int \varphi$, $\int \varphi_1\varphi_2$ etc. instead of $\int_X \varphi(x)\mu(dx)$, $\int_X \varphi_1(x)\varphi_2(x)\mu(dx)$ etc.; we also say »almost everywhere» instead of »μ-almost everywhere».

$\alpha_n \underset{L_p}{\rightharpoonup} \alpha$ will stand for weak convergence in L^p.