A class of hyponormal operators and weak*-continuity of hermitian operators

Kirsti Mattila

We will first in this paper consider a class of hyponormal operators which we call *-hyponormal operators. We give an example of a hyponormal operator which is not *-hyponormal. It follows from a theorem of Ackermans, van Eijndhoven and Martens [1] that subnormal operators on a Hilbert space are *-hyponormal. We prove a generalized Fuglede—Putnam theorem and some other results for these operators.

We will also prove some results on the following problem which was mentioned in [4]:

Problem (1). *Let T be a bounded linear operator on a Banach space X. If $T^*=H+iK$ for some hermitian operators H and K on X^*, is it true that $T=H_0+iK_0$ for some hermitian operators H_0 and K_0 on X?*

It is known that if T^* is normal, then T is normal (Behrends [4]). We show that (1) is true if T^* is a *-hyponormal operator with a weakly compact commutator. Finally we prove that if X is a dualoid space (in particular a dual space) or a C^*-algebra with a unit element, then (1) is true for all operators T such that $T^*=H+iK$.

Let X be a complex Banach space and X^* the dual space of X. We denote by $B(X)$ the space of all bounded linear operators on X. If X and Y are two Banach spaces, then $B(X, Y)$ is the space of all bounded linear operators from X to Y. A *normal* operator on X is an operator which can be written in the form $H+iK$ where H and K are commuting hermitian operators on X. We will only be concerned with bounded operators. The adjoint of an operator $T\in B(X)$ is hermitian if and only if T is hermitian (see [6, § 9] or [7, § 17]). We refer to [6] and [7] for basic facts about numerical ranges and hermitian operators.

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