Spaces of Carleson measures: duality and interpolation

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1. Introduction

In [12], [13] Coifman, Meyer and Stein have developed a theory of "tent spaces" with interesting applications. Their theory has led to a unification and simplification of some basic techniques in harmonic analysis.

The theory of "tent spaces" is closely related to the one of Hardy spaces. In this paper we consider the relationship of these tent spaces with spaces of Carleson measures. In particular we identify the spaces of Carleson measures as the duals of certain tent spaces.

We also compute the real interpolation spaces of some extreme tent spaces by means of computing the corresponding K functionals of Peetre. As pointed out in [13] the interpolation theory of tent spaces can be used to derive the corresponing theory for H^p spaces and Lipschitz spaces.

Let us briefly explain the motivation of this paper (we refer to §2 for detailed definitions). A basic inequality valid for Carleson measures on $\mathbf{R}_+^{n+1}$ is

$$|\mu|\{(x,t)\big||f(x,t)|>\lambda\}\leqq c\big|\{x/A_\infty(f)(x)>\lambda\}\big| \tag{1.1}$$

where $A_\infty(f)(x)=\sup_{\Gamma(x)}|f(y,t)|$, $\Gamma(x)=$cone with vertex x. Then

$$\left|\int f(x,t)\,d\mu(x,t)\right|\leqq c\left(\sup_B\frac{1}{|B|}\int_{T(B)}d|\mu|\right)\|A_\infty(f)\|_1 \tag{1.2}$$

where $T(B)=$"tent with base B".

Let $\|\mu\|_{V^1}=\|C_1(\mu)\|_\infty$, where $C_1(\mu)(x)=\sup_{B\ni x}\frac{1}{|B|}\int_{T(B)}d|\mu|$. Then (1.2) can

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