Adjoint boundary value problems for the biharmonic equation on C^1 domains in the plane

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This is the first of a two-part paper on adjoint problems for the biharmonic equation in a C^1 domain in the plane. The study of problems which arise as adjoints to Dirichlet problems was suggested by the paper of Fabes and Kenig on H^1 spaces of C^1 domains [7]. Their paper shows that the Neumann problem on bounded C^1 domains with h^1 boundary data (see § 1.1 of [7] for details) is solvable as a single layer potential. In addition, the gradient of the single layer potential is used to establish a connection between the h^1 data space and the H^1 space of vectors of harmonic functions in Ω satisfying a generalized Cauchy—Riemann system.

In our papers we show how analogous adjoint problems for the biharmonic equation arise from the potentials and Green's formulae used to study the Dirichlet problem. We give solutions in the form of lower order potentials, a device we introduced in § 5 of [3] to solve the Dirichlet problem.

In the first paper we use the lower order potential to solve the adjoint problems with data in the dual of the space of Dirichlet data. We further show that by considering biharmonic functions as the real parts of solutions of the equation $\bar{\partial}^2 f=0$, $(\bar{\partial}=\partial_x+i\partial_y)$, we are able to solve a fundamental problem in two dimensional elasticity with a modified form of the lower order potential.

The boundary data considered in the first paper is a space of cosets of linear functionals acting on the Dirichlet data. To obtain convergence for the potentials at the boundary we must extend the meaning of the coset space to include functions defined on a system of local parallel translates of the boundary. The trace of the potentials on the boundary is shown to have an inverse in the coset space and a solution of the adjoint problem is obtained.

In the second paper [4] we extend our results to show that for the elasticity problem the potential has non-tangential point-wise limits almost everywhere char-

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