Convergence of complete spline interpolation for holomorphic functions

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1. Introduction

During the 2nd Edmonton conference on approximation theory in June 1982, I. J. Schoenberg stated a conjecture concerning convergence of complete spline interpolation.

Let $S_{2m-1}=S_{2m-1}\left(\frac{1}{n+1}, \dots, \frac{n}{n+1}\right)$ denote the space of spline functions of degree $2m-1$ with simple knots at n equidistant points $\frac{i}{n+1}$, $i=1, \dots, n$, in $(0, 1)$, $S_{2m-1}\subset C^{2m-2}(R)$ and any $S\in S_{2m-1}$ is a polynomial of degree $\leqq 2m-1$ between any two successive knots. The *complete spline interpolation problem* is to find $S(x)\in S_{2m-1}$, where

$$(1.1)\qquad S\left(\frac{\nu}{n+1}\right)=f\left(\frac{\nu}{n+1}\right), \quad \nu = 1, 2, \dots, n$$

$$S^{(i)}(0)=f^{(i)}(0), \quad S^{(i)}(1)=f^{(i)}(1), \quad i=0, 1, \dots, m-1.$$

It is known that (1.1) has a unique solution (see [1]). Concerning this interpolation problem Schoenberg stated

Conjecture 1. *Assume that $f(x)$ is holomorphic in a neighborhood of the interval* $[0, 1]$. *Then there is a fixed value of n depending on f such that*

$$(1.2)\qquad \lim_{m\to\infty} (S_{m,n}f)(x)=f(x)$$

uniformly on $[0, 1]$.

He first raised this conjecture in Budapest in 1968 and again in Oberwolfach in 1971. As a means to study this problem, he also formulated the weaker