On the primary ideal structure at infinity for analytic Beurling algebras

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0. Introduction

An algebra B is a Banach algebra if there is a norm defined on it such that B is a Banach space and the multiplication is continuous. As far as we are concerned, all Banach algebras are assumed commutative. For standard facts in the elementary theory of commutative Banach algebras we refer to [10] and [22].

Suppose w is a locally bounded measurable (weight) function on $\mathbf{R}$, which satisfies

$$\begin{cases} w(x) \geqq 1, & x \in \mathbf{R}, \\ w(x+y) \leqq w(x)w(y), & x, y \in \mathbf{R}. \end{cases} \tag{0.1}$$

Then the space $L^1_w(\mathbf{R})$ of (equivalence classes of) functions f, Lebesgue measurable on $\mathbf{R}$ and satisfying

$$\|f\|_w = \int_{-\infty}^{\infty} |f(x)| w(x)\, dx < \infty$$

is a Banach algebra under convolution multiplication, which we denote by $*$:

$$(f*g)(x) = \int_{-\infty}^{\infty} f(x-t)\, g(t)\, dt \quad \text{for arbitrary} \quad f, g \in L^1_w(\mathbf{R}).$$

Since they were introduced by Beurling in [3], these Banach algebras are called Beurling algebras.

One can show that the limits

$$\alpha = \lim_{x \to +\infty} x^{-1} \log w(x)$$

$$\beta = \lim_{x \to -\infty} x^{-1} \log w(x)$$

are finite, and that $\beta \leqq 0 \leqq \alpha$.