

On the hyperconvexity of holomorphically convex domains in the space \mathbf{C}^n

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§1. Preliminaries

In 1974, Jean-Luc Stehlé has given in his paper [4], such a conjecture¹⁾ that holomorphically convex domain $D = \overset{\circ}{D}$ in \mathbf{C}^n is hyperconvex. In 1976, Jean-Louis Ermine has shown in his paper [1] that this conjecture is positive in case of holomorphically convex Reinhardt domains²⁾. But, in general case, it is as yet unknown that this conjecture is positive or not. Evidently, holomorphically convex domain in \mathbf{C}^n can be approximated by an increasing sequence of analytic polyhedra and analytic polyhedra are hyperconvex.

The purpose of this paper is to give such a proof that this conjecture is positive in case of holomorphically convex domains of some type by means of the above approximation.

Definition 1.³⁾ Let D be a relatively compact open set in \mathbf{C}^n . D is said to be hyperconvex if and only if there exists a plurisubharmonic function $p(z)$ defined on a neighbourhood of \bar{D} and negative on D , such that

$$\{z \in D \mid p(z) \leq c\}$$

is a relatively compact set in D for any $c < 0$.

The following lemma is easily shown from Definition 1.

1) Cf. [4], pp. 167, 177 in which D is relatively compact in \mathbf{C}^n .

2) Cf. [1], pp. 131—133.

3) Cf. [4], p. 163.