A characterisation of Fuchsian groups of convergent type*

N. Th. Varopoulos

Introduction

Let Γ be a Fuchsian group (I make no distinction here between Fuchsian and Frechsoid groups) acting on $U=\{z\in\mathbf{C}, |z|<1\}$ such that every point of ∂U is a limit point of Γ (i.e. Γ is of the first kind. The problem that will be examined does not arise for groups of the second kind). We say that Γ is of convergent type if:

$$\sum_{\gamma\in\Gamma}(1-|\gamma 0|)<+\infty$$

otherwise we say that it is of divergent type.

A group is of convergent type if and only if the corresponding Riemann surface is hyperbolic. It follows therefore that if Γ_0 is as above and is finitely generated then it is of divergent type. Indeed the Riemann surface U/Γ_0 can then be identified with $R\setminus\{r_1, r_2, \ldots, r_k\}$ (for finitely many distinct $r_1, r_2, \ldots, r_k\in R$) and R a compact surface cf. [11], [2].

In this paper I shall consider a subgroup $\Gamma\subset\Gamma_0$ where Γ_0 is finitely generated as above and I shall give a necessary and sufficient condition for Γ to be of convergent type. To state the theorem I shall need some algebraic preliminaries.

Let G be a discrete group generated by a finite number of generators $g_1, g_2, \ldots, g_m\in G$. Let $H\subset G$ be a subgroup and let us fix $\xi_1, \ldots, \xi_k\in G$ finitely many elements of G.

Let us also define $\mu_0, \mu_1, \ldots, \mu_k\in\mathbf{P}(G)$ $k+1$ probability measures on G that are symmetric (i.e. $\mu_j(\{x\})=\mu_j(\{x^{-1}\})$) and which in addition satisfy:

(i) supp μ_0 is finite,

(ii) $\mu_0(g_j)>0$ $j=1, 2, \ldots, m$,

* This research was done while the author was a visitor at the *Mittag-Leffler Institute*. The author is very grateful for the hospitality and the financial support that he received.