Positive solutions of elliptic equations in nondivergence form and their adjoints

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Introduction

We consider uniformly elliptic operators of the form

$$(*) \qquad L = \sum_{i,j=1}^{n} a_{ij}(X)\cdot D^2_{X_i X_j} + \sum_{i=1}^{n} b_i(X)\cdot D_{X_i}$$

with real-valued, bounded measurable coefficients defined in $\mathbf{R}^n$ (for $n \geqq 2$). The functions, a_{ij}, are assumed to be uniformly continuous in $\mathbf{R}^n$ (with no restriction on the modulus of continuity) and satisfy $a_{ij}=a_{ji}$. Operators of this type correspond to diffusion processes in $\mathbf{R}^n$ (see [16]) and hence will be called diffusion operators.

Our main objective is to prove a comparison theorem (Theorem 2.1) for positive solutions of $Lu=0$ in a bounded Lipschitz domain, D, in $\mathbf{R}^n$. The theorem asserts that any two positive solutions of $Lu=0$ in D which vanish on a portion of the boundary must vanish at the same rate. More precisely, if $Q \in \partial D$, $B(8r, Q)$ is a ball of radius $8r$ centered at Q, and u_1 and u_2 are positive solutions of $Lu=0$ in $B(8r, Q)\cap D$ which vanish continuously on ∂D, then

$$\frac{1}{c}\cdot\frac{u_1(X)}{u_1(A_r)} \leqq \frac{u_2(X)}{u_2(A_r)} \leqq c\cdot\frac{u_1(X)}{u_2(A_r)}$$

for all X in $B(r, Q)\cap D$. Here, A_r is a point in $B(r, Q)\cap D$ whose distance from ∂D is proportional to r. The constant, c, is independent of Q, r, u_1, and u_2.

The comparison theorem was proved for harmonic functions in 1968 by Hunt and Wheeden ([9]). It was extended to solutions of $Lu=0$ for operators with Hölder continuous coefficients by A. Ancona in 1978 ([2]). A consequence of the comparison theorem is that the representation theorem and Fatou-type results for positive harmonic functions in D (see [9]) extend to positive solutions of $Lu=0$.

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