On the postulation of canonical curves in $\mathbf{P}^3$

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Introduction

In this paper we consider a very particular problem. We study the postulation of a general smooth curve C in $\mathbf{P}^3$ with $\mathcal{O}_C(1)\cong K_C$, when C has genus 7, 8, 9 or 11.

We call canonical curve in $\mathbf{P}^n$ a smooth, connected curve $C\subset\mathbf{P}^n$ with $\mathcal{O}_C(1)\cong K_C$.

Definition. We say that a subscheme Z of $\mathbf{P}^3$ has maximal rank if foı every integer t the restriction map $\varrho_Z(t):H(\mathbf{P}^3, \mathcal{O}_{\mathbf{P}}(t))\to H^0(Z, \mathcal{O}_Z(t))$ is either injective or surjective.

It is useful to know that a curve C in $\mathbf{P}^3$ has maximal rank since in this case for every k we know completely the dimension of the vector space of surfaces of degree k containing C. For example if C is a canonical curve of genus g with maximal rank, it is contained in a surface of degree k if and only if $\binom{k+3}{3}>(2k-1)(g-1)$.

We prove the following theorem:

Theorem 1. *The general canonical curve of genus* $g=7, 8, 9$ *or* 11 *in* $\mathbf{P}^3$ *has maximal rank.*

The proof is really an existence proof. We construct a reducible curve Z with the expected postulation, i.e. with maximal rank. Then we show that Z is a limit of a flat family of canonical curves in $\mathbf{P}^3$. By semicontinuity a general canonical curve of that genus has maximal rank. The existence of the flat family follows from 2 theorems about degeneration of curves; we will state them in section 1. One of them is due to Hartshorne and Hirschowitz [13]; the other one is contained in [1]. We use often very particular cases of the Brill—Noether theory proved by Griffiths and Harris in [6]. However it seems that sometimes the use of this deep theory could be avoided by ad hoc argument. Theorem 1 is proved separately for each genus $g=7, 8, 9, 11$.

In [7] Gruson and Peskine gave a striking counterexample to a conjecture by Hartshorne [11], showing that no canonical curve of genus 5 or 6 has maximal rank.