Holomorphic mappings with prescribed Taylor expansions

Leif Abrahamsson

1. Introduction

The letters **N** and **C** denote the set of non-negative integers and the set of complex numbers, respectively.

We study the following interpolation problem:

(1) *"Let E and F be two complex, locally convex spaces, Ω an open subset of E and $(z_n)_{n\in\mathbf{N}}$ a sequence of distinct points of Ω. Given any sequence of polynomials $(P_n)_{n\in\mathbf{N}}$ (e.g. continuous polynomials) from E to F, under what conditions on E, F and $(z_n)_{n\in\mathbf{N}}$ does there exist a holomorphic mapping f from Ω to F such that the partial Taylor series of f at z_n up to order $N(n)\geqq\deg P_n$ is equal to P_n for each $n\in\mathbf{N}$?"*

If E and F are one-dimensional, then the answer is that $(z_n)_{n\in\mathbf{N}}$ shall have no accumulation points in Ω. This follows from a combination of Mittag–Leffler's theorem and Weierstrass' theorem.

Weaker versions of (1) have been solved by Y. Hervier [12] and M. Valdivia [24]:

(2) (Y. Hervier [12, Prop. 1, p. 157]). *"Let E and F be two complex Banach spaces and let Ω be a domain of holomorphy in E. Suppose that $(z_n)_{n\in\mathbf{N}}$ is a sequence of distinct elements of Ω such that $\lim_{n\to+\infty}|g(z_n)|=+\infty$ for some holomorphic function g on Ω. Then, given any sequence of elements $(u_n)_{n\in\mathbf{N}}$ of F, there exists a holomorphic mapping f from Ω to F such that $f(z_n)=u_n$ for all $n\in\mathbf{N}$".*

More generally:

(3) (M. Valdivia [24, Thm. 10]). *"Let E be a complex, locally convex space whose topology is given by a family of continuous norms, Ω an open subset of E and F a complex Fréchet space. Suppose that $(z_n)_{n\in\mathbf{N}}$ is a sequence of distinct elements of Ω such that $\lim_{n\to+\infty}|g(z_n)|=+\infty$ for some holomorphic function g on Ω. Then,*