On the growth of subharmonic functions along paths

John Lewis*, John Rossi† and Allen Weitsman*

1. Introduction

Let $\mathbf{C}$ be the complex plane. Then by a path γ tending to ∞, we shall always mean a continuous mapping of $0 \leqq t < 1$ into $\mathbf{C}$ with $\lim_{t\to 1} |\gamma(t)| = +\infty$. If u is subharmonic in $\mathbf{C}$, put $M(r) = M(r, u) = \max_{|z|=r} u(z)$, $0 < r < \infty$. In [7] Huber proved the following theorem:

Theorem A. *Let u be subharmonic in $\mathbf{C}$ and suppose that* $\lim_{r\to\infty} \dfrac{M(r)}{\log r} = +\infty$. *Given $\lambda > 0$ there exists a path, $\Gamma(\lambda)$, tending to ∞ with*

$$\int_{\Gamma(\lambda)} e^{-\lambda u} |dz| < +\infty.$$

In Theorem A, $|dz|$ denotes arc length. Also in [10] Talpur proved

Theorem B. *Let u be subharmonic in $\mathbf{C}$ with* $\lim_{r\to\infty} \dfrac{M(r)}{\log r} = +\infty$. *Then there exists a path Γ tending to ∞ with*

$$\frac{u(z)}{\log |z|} \to \infty \quad as \quad z \to \infty \quad on \quad \Gamma.$$

In this paper, we obtain the following generalization of Theorems A and B, which in fact solves a problem raised by Hayman in [5, p. 12].

Theorem 1. *Let u be subharmonic in $\mathbf{C}$ and suppose that* $\lim_{r\to\infty} \dfrac{M(r)}{\log r} = +\infty$.

† Research carried out at Purdue University while serving as visiting assistant professor. The author gratefully acknowledges the hospitality and support extended him by the Department of Mathematics.

* Research supported by a National Science Foundation grant.