K-divisibility of the K-functional and Calderón couples

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0. Introduction

Let f be a function in $L^p(\mathbf{R}^n)$ with L^p modulus of continuity $\omega(f, t) = \sup_{|h| \leq t} \|f(x+h) - f(x)\|_{L^p}$ satisfying $\omega(f, t) \leq \sum_{n=1}^{\infty} \psi_n(t)$ where $\psi_n(t)$ is a positive concave function of t for each n and $\sum_{n=1}^{\infty} \psi_n(1) < \infty$. Then f can be decomposed into a sum of functions in $L^p(\mathbf{R}^n)$, $f = \sum_{n=1}^{\infty} f_n$ such that, for each n, $\omega(f_n, t) \leq \gamma \psi_n(t)$ for all $t > 0$. (Here γ denotes a constant which does not depend on f.)

This result, which seems far from obvious, is presented as a brief comment in the remarkable note [2] of Ju. A. Brudnyĭ and N. Ja. Krugljak. It is only one of many consequences of their theorem on the property of "K-divisibility" of the Peetre K-functional (Theorem 1 below). Most of the other consequences studied thus far both by Brudnyĭ and Krugljak themselves [2, 3] and by others, notably Per Nilsson [13, 14], are formulated within the context of the theory of interpolation spaces. However, as the above result strongly suggests, it is to be expected that the advances in interpolation theory made possible by the work of Brudnyĭ and Krugljak will also have many further interesting new applications in various branches of analysis (cf. [13], Section 6.2).

In the present paper we present an alternative proof of the K-divisibility theorem and subsequently, with the help of the techniques developed in our proof, we show that all interpolation spaces with respect to a large class of compatible couples $\bar{A} = (A_0, A_1)$ have certain monotonicity properties with respect to the K-functional. Among the corollaries of our main theorem we obtain the result of Sparr [16] and its generalization due to Dmitriev [8] characterizing all interpolation spaces with respect to couples of L^p spaces. But now we can also give a weaker form of Dmitriev's theorem which holds for values of the exponents p for which the original version breaks down. Another corollary describes monotonicity conditions satisfied

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