The local Π_1 of the complement of a hypersurface with normal crossings in codimension 1 is abelian

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0. Introduction

Let $(X, 0)\subset(\mathbf{C}^{n+1}, 0)$ be a germ of reduced analytic hypersurface in $(\mathbf{C}^{n+1}, 0)$ defined by $f=0$, where $f\in\mathcal{O}_{\mathbf{C}^{n+1},0}$ is a germ of analytic function in $\mathbf{C}^{n+1}$ at 0. We shall prove the following:

Main Theorem. *Assume that outside an analytic subgerm $(Y, 0)$ of $(X, 0)$ of dimension at most $n-2$ the only singularities of $(X, 0)$ are normal crossings then the local fundamental group of the complement of $(X, 0)$ in $(\mathbf{C}^{n+1}, 0)$ is abelian.*

Remark. Using Milnor fibration theorem ([M] theorems 4.8. and 5.11.) this theorem implies that under its hypothesis the Milnor fiber of $(X, 0)$ has a fundamental group which is free abelian of rank the number of analytic components of X at 0 minus one. In particular, if $(X, 0)$ is analytically irreducible, the Milnor fiber is simply connected. This result extends a result of M. Kato and Y. Matsumoto ([K—M]) which says that if the singular locus of $(X, 0)$ has codimension 2, the Milnor fiber of $(X, 0)$ is simply-connected.

We shall still denote by X and Y representants of $(X, 0)$ and $(Y, 0)$ in a sufficiently small neighbourhood of 0 in $\mathbf{C}^{n+1}$.

We notice that, if $\varepsilon>0$ is small enough, the balls B_ε^{2n+2} of $\mathbf{C}^{n+1}$ centered at 0 with radius $\varepsilon>0$:

$$B_\varepsilon^{2n+2}:=\{z\in\mathbf{C}^{n+1}, \|z\|\leq\varepsilon\}$$

— make a fundamental system of good neighbourhoods of 0 in $\mathbf{C}^{n+1}$ with regard to both X and Y in the sense of D. Prill (cf. [P] definition 1) by using the local conic structure of an analytic set (cf. [B—V] lemma (3.2.)).