Curve singularities of finite Cohen—Macaulay type

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Let R be a reduced local Noetherian ring of dimension one, and assume that the integral closure $\tilde{R}$ is finitely generated as an R-module. The main theorem of [13] gives necessary and sufficient conditions in order that R have only finitely many non-isomorphic indecomposable finitely generated torsionfree modules. These necessary and sufficient conditions (listed in (1.2)) were introduced by Drozd and Roiter in [2] and shown to be equivalent to finite CM type for localizations of orders in algebraic number fields. (See also [8] and [5].) Since the non-zero finitely generated torsionfree modules are exactly the maximal Cohen—Macaulay modules, these rings are said to have *finite CM type*. In the geometric case, when R is the local ring of a singular point of an algebraic curve over a field k, these conditions impose stringent conditions on the singularity: Its multiplicity must be less than or equal to three, and when the multiplicity is three there is an additional condition, harder to describe in geometric terms. While this latter condition is easy to test for any specific singularity, it seems worthwhile to give an explicit classification, up to analytic isomorphism, of those singularities whose local rings have finite CM type.

In their 1985 paper [4] Greuel and Knörrer gave explicit equations for the plane curve singularities of finite CM type over an algebraically closed field of characteristic zero. The classification in positive characteristics (but still over an algebraically closed field) was obtained by Kiyek and Steinke in [99]. The classification given here (in § 5) is valid over arbitrary fields and includes space curves as well as plane curves.

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