

Pointwise regularity of solutions to nonlinear double obstacle problems

Tero Kilpeläinen¹ and William P. Ziemer²

1. Introduction

In this paper we investigate the continuity at a given point x_0 of the solution u of a nonlinear elliptic variational inequality with a double obstacle constraint of the form $\psi_1 \leq u \leq \psi_2$. The partial differential operators associated with our obstacle problem are quasi-linear and include operators with only L^∞ coefficients. The obstacles in this context are to be regarded as quite general and irregular. In particular, they may be discontinuous. Since the partial differential operators associated with our problem may have only L^∞ coefficients, we can expect at most Hölder continuity for the regularity of our solution. Indeed, we show that if both obstacles are locally Hölder continuous, then the solution is also locally Hölder continuous. We also show that if the obstacles are not continuous, but satisfy a Wiener-type regularity condition, the solution is still continuous. This work extends that of [MZ1] in which a similar investigation was undertaken for the case of a single obstacle. This work also extends the recent paper of [DMV] which is devoted to the double obstacle problem for linear operators with bounded measurable coefficients. Because their work involves linear operators, they are able to employ potential theoretic techniques to obtain many of their estimates. These techniques are not available for us in our context of nonlinear operators.

Since the one-obstacle problem is a special case of the two-obstacle problem, one cannot expect better results in this latter case. On the other hand, the two-obstacle problem is so similar to the unilateral case, that one would anticipate virtually identical results. However, we have not been able to achieve this with the general structure we consider. One significant difference between the single and

¹ Research conducted in part while visiting Indiana University.

² Research supported in part by a grant from the National Science Foundation.