Weighted L^p estimates for oscillating kernels

Sagun Chanillo, Douglas S. Kurtz and Gary Sampson

0. Introduction

Let $0<a\neq 1$, $b\leqq 1$, $y\in\mathbf{R}$, and consider the kernels

$$K_{a,b+iy}(t) = \exp(i|t|^a)(1+|t|)^{-(b+iy)}, \quad t\in\mathbf{R}.$$

The convolution operators $K_{a,b+iy}*f$ and closely related weakly singular operators and multiplier operators have been studied by many authors. It is well known that these operators satisfy the following norm inequalities [5, 8, 10, 12, 13, 14]:

$$\|K_{a,1}*f\|_p \leqq C\|f\|_p, \quad 1<p<\infty \tag{0.1}$$

and

$$\|K_{a,b}*f\|_p \leqq C\|f\|_p, \quad b<1, \quad \frac{a}{2}+b\geqq 1, \quad \frac{a}{a+b-1}\leqq p\leqq \frac{a}{1-b}. \tag{0.2}$$

In addition, $K_{a,1}$ maps H^1 into L^1 and, by duality, L^∞ into BMO.

The purpose of this paper is to consider norm inequalities of the form

$$\|K_{a,b+iy}*f\|_{p,w} \leqq C\|f\|_{p,w},$$

where $\|f\|_{p,w}=(\int_{\mathbf{R}}|f(x)|^p w(x)\,dx)^{1/p}$. Our approach is to consider these operators as convolutions, although they can be treated as multipliers and many of our results originated from this latter point of view.

Our first result is

Theorem 1. *Let* $0<a\neq 1$, $b\leqq 1$, *and* $\frac{a}{2}+b\geqq 1$. *Let* $1<p<\infty$ *when* $b=1$ *and* $\frac{a}{a+b-1}\leqq p\leqq\frac{a}{1-b}$ *when* $b<1$. *Let* $w\in A_p$ *and define* $\alpha=a\left|\frac{1}{p}-\frac{1}{2}\right|+1-\frac{a}{2}$ *and* $\delta=\frac{b-\alpha}{1-\alpha}$. *Then,*

$$\|K_{a,b+iy}*f\|_{p,w^\delta} \leqq C(1+|y|)\|f\|_{p,w^\delta}.$$