On Besov, Hardy and Triebel spaces for $0<p\leqq 1$

Bui Huy Qui

Introduction

The aim of the present paper is two-fold. Our first aim is to derive a Hardy—Littlewood type characterization for non-homogeneous Besov spaces defined by Peetre [13] in the case $0<p<1$ via traces of temperatures on the upper halfspace R_+^{n+1}, and thus we answer a question related to the one raised by Peetre [15; p. 258, Remark]. This characterization completes the work of Taibleson [16] and Flett [5] in the case $1\leqq p\leqq\infty$, and may be also considered as non-periodic version of another result of Flett [6]. The idea of the proof comes from the classical work of Gwilliam [9] as was done by Peetre [15] in a characterization of homogeneous spaces via harmonic functions (cf. also [6]); other tools are a sub-meanvalue property of temperatures proved in section 1, which is similar to a result of Hardy—Littlewood given in the paper of Fefferman—Stein [4], and results from interpolation theory. As a consequence of this characterization, we extend some results on translation invariant operators on Besov and Hardy spaces to the case $0<p\leqq 1$ (cf. [2], [12], [15]).

Our second (and main) aim concerns pseudo-differential operators. In [15], Peetre showed that if $\sigma\in C^\infty(R^n\times R^n)$, $1\leqq p\leqq\infty$ and

$$(1)\qquad |D_x^\alpha D_\xi^\beta\sigma(x,\xi)|\leqq C_{\alpha,\beta}(1+|\xi|)^{-|\beta|},$$

then the associated pseudo-differential operator $T=\sigma(D)$ is bounded on $B_{p,q}^s(-\infty<s<\infty,\ 0<q\leqq\infty)$. As for the case $0<p<1$, he required a "somewhat stronger" assumption on the symbol (cf. [15; pp. 285—287]); however, it is not difficult to prove that T is still bounded in this case under the same condition (1). On the other hand, Gibbons [7] has proven the boundedness of T on $B_{p,q}^s(0<s<1,\ 1\leqq p,\ q\leqq\infty)$ under the following assumption on the symbol:

$$(2)\qquad \|D_\xi^\beta\sigma(\cdot,\xi)\|_{B_{\infty,q}^s}\leqq C_\beta(1+|\xi|)^{-|\beta|}.$$