On the lattice points on unicursal cubic curves

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The theory of rational points on a plane unicursal curve

$$f(x, y, z) = 0 \tag{A}$$

and their determination through a parametric representation is essentially complete due to the work of Poincaré, the special case of the conic having been considered earlier by Gauss in the *Disquisitiones Arithmeticae.* This theory, however, does not answer fully the problem of determining all integer solutions of (A) through an algebraic parametric representation, since, as Cantor [1] remarked in respect of the conic, the transition from a rational solution to a corresponding integral solution may lead to the latter being affected by a common factor that is not directly expressible algebraically. Having in a previous paper [2] discussed Cantor's remark and obtained in the case of the conic a parametric representation for the integer solutions of (A) in terms of triplets of quadratic forms with invariants related to f, we turn in the present communication to the corresponding problem for the unicursal cubic curve. By a method more geometrical in nature than that used in [2] but applicable in principle to unicursal curves of any degree we shew that for the unicursal cubic curve there is also a complete parametric representation of the integer solutions of (A) by a set of triplets of binary forms, these being now of degree 3.

The theory is analogous to that for the conic in that firstly the invariants of the representing triplets are related to the coefficients of f and in that secondly each primitive solution, except that corresponding to the double point, is obtained precisely once. Moreover, again, triplets with given invariants belong to a finite number of classes. On the other hand the theory of the class number contains features that are not presented in the quadratic case or indeed in most situations relating to homogeneous forms, there being for example the fact that for any given (possible) invariant system there exists a proportional invariant system for which

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