Sets of synthesis and sets of interpolation for weighted Fourier algebras

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§ 1. Introduction. Let $A_0(\mathbf{T})$ denote the Banach algebra of continuous functions with absolutely convergent Fourier series. We define

$$A_\alpha(\mathbf{T}) = \{f \in C(\mathbf{T}) : \sum_n |\hat{f}(n)|\,(1 + |n|)^\alpha < \infty\},\ \ \alpha > 0\,.$$

We shall also be concerned with the Banach algebra of Lipschitz functions $\Lambda_\alpha(\mathbf{T}), \lambda_\alpha(\mathbf{T})$ and $(\lambda_\alpha \cap A)(\mathbf{T})$. We let $\lambda_0(\mathbf{T}) = C(\mathbf{T})$ and $\lambda_1(\mathbf{T}) = C^1(\mathbf{T})$, [6; pp. 42—43].

Let $R \subset C(\mathbf{T})$ be a regular Banach algebra such that the maximal ideal space of R is $\mathbf{T}$. For a closed subset E of $\mathbf{T}$, we define

$$I^R(E) = \{f \in R : f = 0 \ \text{ on } \ E\}\,,$$

$R(E) = R/I^R(E)$ is the restriction algebra of R to E.

$$\tilde{R}(E) = \left\{f \in C(E) : \sup_{\substack{\mu \in M(E)\\ \mu \neq 0}} \frac{|\int f\,d\mu|}{\|\mu\|_{R'}} < \infty\right\}$$

where R' is the dual of R. $\tilde{R}(E)$ is called the tilda algebra of $R(E)$. For $f \in \tilde{R}(E)$, $\|f\|_{\tilde{R}}$ is defined by

$$\|f\|_{\tilde{R}} = \sup_{\substack{\mu \in M(E)\\ \mu \neq 0}} \frac{|\int f\,d\mu|}{\|\mu\|_{R'}}\,.$$

Let I be a closed ideal in R, then hull I is defined to be the set of common zeros of all functions in I. We say that a closed subset E of $\mathbf{T}$ is of *synthesis* in R if $I^R(E)$ is the only closed ideal in R whose hull is E and that *ideal theorem holds for* E in R if every closed ideal I in R whose hull is E is the intersection of all closed primary ideals containing I. We let