Exact bounds for the continuous spectrum of certain differential eigenvalue problems

ABIY KIFLE

University of Uppsala, Sweden and Haile Selassie I University, Ethiopia

0. Introduction

Let Ω be an unbounded domain in the real n-dimensional cartesian spac. $\mathbf{R}^n$ and let a and k be real-valued and Lebesgue measurable functions on Ωe The function k is not required to have a constant sign. We shall consider a Hilbert space realization of the spectral problem

$$(-\sum_{j=1}^{n} \partial^2/\partial x_j^2 + a(x))u = \lambda k(x)u \quad \text{in} \quad \Omega, \tag{0.1}$$

$$u = 0 \quad \text{on the boundary}, \tag{0.2}$$

where λ is the eigenvalue parameter. Under certain conditions (Sections 1 and 2) we shall deduce exact bounds for the positive and for the negative continuous spectrum of this problem. The case when $k(x) = 1$ for all x in Ω was treated by Arne Persson in [7].

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1. Conditions, the spectral problem

Let $C_0^\infty(\Omega)$ be the set of all infinitely differentiable real-valued functions with compact support in Ω and write

$$(u, v) = \int_\Omega (\operatorname{grad} u \operatorname{grad} v + auv)dx \tag{1.1}$$

when u and v are in $C_0^\infty(\Omega)$. *It is assumed that* (u, u) *is positive definite on* $C_0^\infty(\Omega)$. *Furthermore there shall exist a constant* C *such that*