A problem on the union of Helson sets

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Let G be a locally compact abelian group and let $\hat{G}$ be its dual group.

Definition 1. A compact subset $E \subset G$ is called *Kronecker* if for every continuous function f on E of modulus identically one $(|f(x)| \equiv 1, \ \forall\, x \in E)$ and for every $\varepsilon > 0$ there exists $\chi \in \hat{G}$ such that

$$\sup_{x \in E} |f(x) - \chi(x)| \leq \varepsilon \qquad \text{(cf. [1] ch. 5, § 1)}.$$

We shall denote by $M(G)$ the set of all bounded complex valued Radon measures on G and by $M(E)$ the elements of $M(G)$ with support in a compact subset E of G.

We shall denote by $C(E)$ the set of all continuous complex valued functions on E.

Definition 2. A compact subset E of G is called a *Helson α-set* (*H_α-set*) if there exists a constant $\alpha > 0$ such that

$$\|\hat{\mu}\|_\infty = \sup_{\chi \in \hat{G}} |\hat{\mu}(\chi)| \geqq \alpha \, \|\mu\|$$

for every $\mu \in M(E)$, (observe that then $0 < \alpha \leq 1$).

If K is a compact subset of G we shall write $\mathrm{Gp}\,(K)$ for the group generated by K in G.

In this paper we shall prove the following theorem.

Theorem. *Let K be a totally disconnected Kronecker subset of G and D a countable compact H_α-subset of G such that*

$$\mathrm{Gp}\,(K) \cap \mathrm{Gp}\,(D) = \{0\}\,.$$

Then $K \cup D$ is an H_α-set.