The center and the commutator subgroup in hopfian groups

R. Hirshon

Polytechnic Institute of Brooklyn, Brooklyn, N.Y., U.S.A.

1. Abstract

We continue our investigation of the direct product of hopfian groups. Throughout this paper A will designate a hopfian group and B will designate (unless we specify otherwise) a group with finitely many normal subgroups. For the most part we will investigate the role of $Z(A)$, the center of A (and to a lesser degree also the role of the commutator subgroup of A) in relation to the hopficity of $A \times B$. Sections 2.1 and 2.2 contain some general results independent of any restrictions on A. We show here

(a) If $A \times B$ is not hopfian for some B, there exists a finite abelian group F such that if k is any positive integer a homomorphism θ_k of $A \times F$ onto A can be found such that θ_k has more than k elements in its kernel.

(b) If A is fixed, a necessary and sufficient condition that $A \times B$ be hopfian for all B is that if θ is a surjective endomorphism of $A \times B$ then there exists a subgroup B_* of B such that $A\theta B = A\theta \times B_* \theta$.

In Section 3.1 we use (a) to establish our main result which is

(c) If all of the primary components of the torsion subgroup of $Z(A)$ obey the minimal condition for subgroups, then $A \times B$ is hopfian.

In Section 3.3 we obtain some results for some finite groups B. For example we show here

(d) If $|B| = p^e q_1^{e_1} \dots q_s^{e_s}$ where $p, q_1 \dots q_s$ are the distinct prime divisors of $|B|$ and if $0 \le e \le 3$, $0 \le e_i \le 2$ and $Z(A)$ has finitely many elements of order p^2 then $A \times B$ is hopfian.

Several results of the same nature as (*d*) are obtained here.

In Section 4 we obtain some results similar to (d) by placing some restrictions on the commutator subgroup of A. We also show here

(e) $A \times B$ is hopfian if B is a finite group whose Sylow p subgroups are cyclic.

(f) $A \times B$ is hopfian if B is a perfect group.