# On the local and global non-characteristic Cauchy problem when the solutions are holomorphic functions or analytic functionals in the space variables

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## 1. Introduction

In [5], p. 116, L. Hörmander proves the linear Cauchy-Kovalevsky theorem by a method of successive approximations. The main lemma used by Hörmander in that proof is also found in M. Nagumo [6]. Nagumo uses the lemma and Schauder's fixed point theorem to prove the Cauchy-Kovalevsky theorem. L. V. Ovsjannikov [7] has used the ideas in the lemma to prove a theorem that could be called an abstract Cauchy-Kovalevsky theorem. We shall call it the Ovsjannikov theorem. The theorem treats a Cauchy problem for an ordinary differential equation for functions with values in certain Banach spaces that form a scale of Banach spaces. See also F. Treves [22] and [23]. The Ovsjannikov theorem can be used to prove the linear Cauchy-Kovalevsky theorem as is done in [7], [22], and [23].

In [23], p. 24, Treves proves the dual Ovsjannikov theorem taking as scale of Banach spaces the duals of the spaces in the original scale. Then on pp. 53—58 [23] Treves takes the dual of the scale used by Ovsjannikov to prove the linear Cauchy-Kovalevsky theorem and applies the dual Ovsjannikov theorem. This gives the dual Cauchy-Kovalevsky theorem. Here the coefficients are analytic functions just as in the ordinary theorem but the solution is a function of the time variable with values in the space of analytic functionals on the space of analytic functions of the space variables.

The purpose of this paper is the following. We shall give another proof of the dual Cauchy-Kovalevsky theorem, Theorem 1. We shall also prove a global version of that theorem, Theorem 2, together with a global version of the ordinary Cauchy-Kovalevsky theorem, Theorem 3. In the proofs of the dual theorems we shall use the Fourier-Borel transformation of analytic functionals. Then the dual theorems are transformed into theorems for partial differential equations of infinite order in the space variables. The solutions of the transformed problems