Hyperfinite product factors

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1. Introduction

It is an open question whether all hyperfinite factors are *-isomorphic to factors obtained as the infinite tensor product of finite type I factors. In order to study this problem it is necessary to have criteria which tell us when a hyperfinite factor is *-isomorphic to such a product factor. The present paper is devoted to a result of this kind, the criterion being that all, or equivalently, just one normal state is in a sense asymptotically a product state. This result is an intrinsic characterization of product factors in that it is independent of any weakly dense UHF-algebra and also of any tensor product factorization of the underlying Hilbert space.

We first recall some terminology. A UHF-algebra is a C^*-algebra $\mathfrak{A}$ with identity I in which there is an increasing sequence of I_{n_i}-factors M_{n_i} containing I such that $n_i \to \infty$ and $\bigcup_{i=1}^{\infty} M_{n_i}$ is uniformly dense in $\mathfrak{A}$, see [2]. A factor $\mathfrak{R}$ is said to be hyperfinite if there is a UHF-algebra which is weakly dense in $\mathfrak{R}$. More specially $\mathfrak{R}$ is said to be an ITPFI-factor (infinite tensor product of finite type I factors) if there exists an infinite sequence of I_{n_i}-factors M_{n_i} with $n_i \geq 2$ for an infinite number of i's, and a product state $\omega = \otimes_{i=1}^{\infty} \omega_i$ of the C^*-algebraic tensor product $\mathfrak{A} = \otimes_{i=1}^{\infty} M_{n_i}$, such that $\mathfrak{R}$ equals the weak closure of $\pi_\omega(\mathfrak{A})$, where π_ω is the representation of $\mathfrak{A}$ induced by ω. It was shown by Murray and von Neumann, see [1, Théorème 3, p. 280], that all hyperfinite II_1-factors are *-isomorphic, and hence *-isomorphic to ITPFI-factors. It is not known whether all hyperfinite factors of types II_∞ or III are *-isomorphic to ITPFI-factors. We refer the reader to the book of Dixmier [1] for the theory of von Neumann algebras and to the paper of Guichardet [3] for that of infinite tensor products.

The author is indebted to J. Tomiyama for pointing out a gap in an early version of the paper. In this version there was also a rather long proof of the implication