Convergence almost everywhere of certain singular integrals and multiple Fourier series

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Introduction

We shall here study certain several variable analogues of the operator M^* defined by

$$M^*f(x) = \sup_n \left| \int_{-\pi}^{\pi} \frac{e^{-int}f(t)}{x-t}\, dt \right| , \quad |x| \leq \pi ,$$

and the Fourier series maximal operator treated by Carleson [4] and Hunt [7].

Let $\mathbf{R}^s$ be the Euclidean space of dimension s and let $T_s = \{x = (x_1, \ldots, x_s) \in \mathbf{R}^s;\ 0 \leq x_i \leq 2\pi,\ i = 1, 2, \ldots, s\}$. If $x = (x_1, \ldots, x_s)$ and $\xi = (\xi_1, \ldots, \xi_s)$ belong to $\mathbf{R}^s$ we set $x \cdot \xi = \sum_{i=1}^{s} x_i \xi_i$ and $|x| = \left(\sum_{i=1}^{s} x_i^2 \right)^{1/2}$.

L. Hörmander has observed that the first part of the proof in [4] can be generalized to yield the following (unpublished) result.

If k is a C^∞ Calderón — Zygmund kernel defined in $\mathbf{R}^s$, $s \geq 2$, and if $\int_{T_s} |f(x)|\, (\log^+ |f(x)|)^{1+\delta}\, dx < \infty$ for some $\delta > 0$, then $\left| \int_{T_s} k(x-t) e^{-i\xi \cdot t} f(t) dt \right| = o(\log \log |\xi|)$, $|\xi| \to \infty$, for almost every x in T_s.

In Sections 1 to 3 in this paper we prove among other things the following theorem, which generalizes the L^p estimate of the operator M^* in [7].

THEOREM. *Assume that k is a Calderón — Zygmund kernel defined in $\mathbf{R}^s$, $s \geq 2$, which has continuous derivatives of order $\leq s+1$ outside the origin. Let the operator M be defined by*

$$Mf(x) = \sup_{\xi \in \mathbf{R}^s} \left| \int_{T_s} k(x-t)\, e^{-i\xi \cdot t} f(t) dt \right| , \quad x \in T_s .$$