Cohomology of operator algebras II. Extended cobounding and the hyperfinite case

RICHARD V. KADISON and JOHN R. RINGROSE

University of Pennsylvania, Philadelphia, Pennsylvania, U.S.A.

and

University of Newcastle upon Tyne, England

1. Introduction

We continue the programme begun in [1] of studying the (topological) cohomology of operator algebras. In that article, we proved that cohomology of a type I von Neumann algebra with coefficients in the algebra vanishes [1: Theorem 4.4]. Employing that theorem and the various preparatory results on centre adjustment of cocycles, we prove (Theorem 2.4), in this paper, that each cocycle on a (general) von Neumann algebra with coefficients in the algebra cobounds a cochain with coefficients in the algebra of all bounded operators on the Hilbert space on which the von Neumann algebra acts. This theorem is, then, used to prove (Theorem 3.1) that cohomology with coefficients in the algebra vanishes for hyperfinite von Neumann algebras.

The argument proving Theorem 2.4 is structurally the same as that appearing in [4; Theorem 4]. It is made more difficult by the fact that higher-dimensional (norm-continuous) cocycles do not satisfy automatic weak continuity conditions (as do derivations [4; Lemma 3]). This same difficulty rules out certain direct approaches to dealing with the hyperfinite case.