Holomorphic convexity and analytic structures in Banach algebras

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Introduction

In this paper every Banach algebra is commutative and semi-simple with a unit element. If B is a Banach algebra then M_B denotes its maximal ideal space and ∂_B its Shilov boundary. Since B is semi-simple the Gelfand transform identifies B with a subalgebra of $C(M_B)$. If B then becomes a uniformly closed subalgebra of $C(M_B)$ we say that B is a uniform algebra. Notice that B is a uniform algebra precisely when it is complete in its spectral radius norm.

If B is a uniform algebra and if A is a closed point-separating subalgebra of $C(M_B)$ such that $A \subset B$, then we simply say that A is a *subalgebra of* B. In Section 1 and 2 we analyze this situation. Here follow two results which are typical applications of this material.

THEOREM 1.1. *Let X be a reduced analytic space and let A be a point-separating subalgebra of $\mathcal{O}(X)$. Suppose that K is a compact $\mathcal{O}(X)$-convex set in X such that the set $\hat{K}_A = \{x \in X : |f(x)| \leq |f|_K$ for all f in $A\}$ is compact in X. Then $K = \hat{K}_A$ and $M_{A(K)} = K$, where $A(K)$ is the uniform algebra on K generated by the restriction algebra $A|K$.*

In Theorem 2.1. the following notations are used. D is the closed unit disc in $\mathbf{C}^1$ and $A(D)$ is the usual disc-algebra on D. An element $f \in A(D)$ is *smooth* if the restriction of f to T is continuously differentiable. Here T is the unit circle.

THEOREM 2.1. *Let A be a subalgebra of $A(D)$ such that A contains a dense subalgebra of smooth functions. Then $M_A = D$.*

Let us remark here that the results above also apply to Banach algebras. For if B is a Banach algebra and if B_c is the uniform closure of B in $C(M_B)$, then it is well known that $M_B = M_{B_c}$. If A is a closed subalgebra of B which separates points in M_B, then A_c is a subalgebra of B_c.