Some remarks on the value distribution of meromorphic functions

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1. Introduction

1. Let E be a closed set in the complex plane and f a meromorphic function outside E omitting a set F. We shall consider the following problem: If E is thin, under what conditions is F thin, too? In Chapter 2 we consider the case when E and F are of Hausdorff dimension less than one. In Chapter 3 E and F are countable sets with one limit point, and in Chapter 4 E is a countable set whose points converge to infinity, f is entire, and F is allowed to contain at most one finite value.

2. Sets of dimension less than one

2. Let f be meromorphic and non-constant outside a closed set E in the complex plane. It is known that if the logarithmic capacity of E is zero then f cannot omit a set of positive capacity, and if E has linear measure zero then f cannot omit a set of positive $(1+\varepsilon)$-dimensional measure. If the dimension of E is greater than one then there exists a non-constant function f which is regular and bounded outside E. Carleson [1] has proved that there exists a set E of positive capacity such that if f omits 4 values outside E then f is rational. We consider the following problem: Let E be of dimension less than one. Can f omit a set whose dimension is greater than the dimension of E?

We denote by $\mathrm{Dim}\,(A)$ the Hausdorff dimension of a set A, and let $\dim\,(A)$ be the dimension of A obtained by using coverings consisting of discs with equal radii. For example for usual Cantor sets these dimensions are equal. We have the following answer to our question:

THEOREM 1. *Let E be a closed set with $\dim\,(E) < 1$. If f is meromorphic and non-constant outside E and omits F then* $\mathrm{Dim}\,(F) \leq \dim\,(E)$.

The proof will be given in 3 and 4.

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