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Continuity of pluricomplex Green functions with poles along a hypersurface

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I. Introduction

Let Ω be a bounded domain in $\mathbf{C}^n$ and A be a complex hypersurface of Ω. Following [LS2] we define

$$\mathcal{F}_{A,\Omega}=\{u\in \operatorname{PSH}(\Omega): u\leq 0 \text{ and } \nu_u(a)\geq \nu_A(a) \text{ for all } a\in A\},$$

and

$$G_{A,\Omega}(z)=\sup_{u\in\mathcal{F}_{A,\Omega}} u(z),$$

where $\operatorname{PSH}(\Omega)$ is the class of plurisubharmonic functions on Ω (including the constant function $-\infty$), $\nu_u(a)$ denotes the Lelong number of u at a and $\nu_A(a)$ denotes the multiplicity of A at a. Recall also that, if u is plurisubharmonic in a neighborhood of $a\in\mathbf{C}^n$ then

$$\nu_u(a)=\lim_{r\to 0}\frac{\sup_{|z-a|=r} u(z)}{\log r}.$$

The pluricomplex Green function $G_{A,\Omega}$ was first introduced in a more general setting by F. Lárusson and R. Sigurdsson in [LS2]. In the same paper, the authors studied boundary behavior and uniqueness of $G_{A,\Omega}$. They also discussed the relationship between $G_{A,\Omega}$ and disc functionals and their envelopes (see also [LS1]). In particular, the following result about the "almost" continuity of $G_{A,\Omega}$ was claimed in Theorem 3.9 of [LS2]: *Let X be a relatively compact domain in a Stein manifold with a strong plurisubharmonic barrier at every boundary point. Let A be the divisor of a holomorphic function f on X which extends to a continuous function on $\overline{X}$. Then the set of points in X where $G_{A,\Omega}$ is discontinuous is pluripolar.*

Unfortunately, it turns out as reported in [LS3], that there is a gap in the proof of this statement. It is not known whether this "theorem" is valid even in the case where X is a ball in $\mathbf{C}^n$ and A is an (arbitrary) divisor (see also the remark after