THE MORI PROPERTY IN RINGS WITH ZERO DIVISORS, II

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ABSTRACT. A commutative ring $R$ is said to be a Mori ring if it satisfies the ascending chain condition on regular divisorial ideals. Contrary to what happens in Mori domains, examples exist which show if $P$ is a prime ideal of a Mori ring $R$, then $R_P$ need not be a Mori ring. However, if the total quotient ring of $R$ is von Neumann regular, then it is the case that $R_P$ is Mori whenever $R$ is Mori. In fact, when the total quotient ring is von Neumann regular, then $R$ is a Mori ring if and only if $R_P$ is a Mori ring for each maximal $t$-ideal $P$ and each regular nonunit of $R$ is contained in at most finitely many maximal $t$-ideals.

1. Introduction. An integral domain $D$ is said to be a Mori domain if it satisfies the Ascending Chain Condition (ACC) on divisorial ideals [23]. Following the terminology introduced in [19], we say that a ring $R$ is a Mori ring if it satisfies ACC on regular divisorial ideals, regular meaning each ideal contains an element that is not a zero divisor. There are several conditions which are known to be equivalent to a domain $D$ being Mori including having $D$ satisfy the Descending Chain Condition on those descending chains of divisorial ideals whose intersection is nonzero [26, Théorème I.1] and having $D_M$ a Mori domain for each maximal $t$-ideal $M$ with each nonzero nonunit a unit in all but finitely many such localizations [26, Théorème I.2], [24, Théorème 2.1]. Theorem 2.22 of [19] shows that $R$ is a Mori ring if and only if it satisfies DCC on those descending chains of regular divisorial ideals whose intersections are regular ideals. One of our main purposes here is to extend the local characterization of Mori domains to reduced rings whose total quotient rings are von Neumann regular. We shall also consider the problem of determining when the polynomial ring $R[x]$ is a Mori ring.

In this paper $R$ will always denote a commutative ring with identity. Also, we use $T(R)$ to denote the total quotient ring of $R$ and $Q_0(R)$ to