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A MATHEMATICAL THEORY OF QUANTUM COHOMOLOGY

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1. Introduction

Topological σ models, proposed by Witten [25], have become increasingly important in string theory and many of its important applications like quantum cohomology and mirror symmetry. But Witten proposed it based on physical intuition. Until recently, its rigorous mathematical foundation remained to be established. The first step was taken by the first author in [19], where he established the mathematical definition of topological σ model invariant, *k-point correlation function*, for rational curves. One of the main features in [19] is, predicted by Witten, the use of symplectic topology, in particular, of pseudo-holomorphic curves. As Witten pointed out [25], the topological σ model is a $1+1$ topological field theory. A key topological field theory axiom is the composition law. In this paper, we will first define a *mixed invariant* for arbitrary genus, which combines the topological σ-model invariant with the Gromov invariant. Such a mixed invariant is natural in considering the composition law of the Gromov invariant. The main part of this paper is to give a mathematical proof of the composition law of our mixed invariant, which includes the topological σ model invariant. There are many applications of this composition law. The obvious one is to compute any k-point correlation function in terms of 3-point functions. In this paper, we will give three other important applications. The first application is a mathematical proof of the existence of quantum ring structures on cohomology groups of semi-positive symplectic manifolds. The existence of quantum ring structures was first sug-

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